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70 EXERCISES.

for the equation to the tangent to the hyperbola,

$$xx' - p(x'y + y'x) - yy' = c(I - m)$$

or

$$y = \frac{x' - py'}{y' + px'} x - \frac{c(1 - m)}{y' + px'}.$$
 (6)

Let  $\alpha$  equal the angle between the central radius and the tangent to the ellipse, and  $\beta$  equal the angle between the tangents to the ellipse and the hyperbola; then

$$\tan \beta = \frac{x'y' - py'^2 + mx'y' + mpx'^2}{y'^2 + px'y' - mx'^2 + mpx'y'},$$
  
$$\tan \alpha = \frac{mx'^2 + y'^2}{(m-1)x'y'}.$$

That tan  $\alpha$  may equal tan  $\beta$ , we must have, discarding the primes,  $(mx^2 + y^2)(y^2 + pxy - mx^2 + mpxy) = (mxy - xy)(xy - py^2 + mxy + mpx^2)$ , which reduces to

$$y^4 + 2mpx^3y + 2mpxy^3 - m^2x^4 - m^2x^2y^2 + x^2y^2 = 0,$$
 (7)

or, substituting for y its value from (1),

$$m^{2}(c-x^{2})^{2} + 2mpx^{3}\sqrt{m(c-x^{2})} + 2m^{2}px(c-x^{2})\sqrt{m(c-x^{2})},$$

$$-m^{2}x^{4} - m^{3}x^{2}(c-x^{2}) + mx^{2}(c-x^{2}) = 0.$$
(8)

From (3),

$$2px\sqrt{m(c-x^2)} = (1+m)x^2 - c.$$

Substituting in (8), we get

$$m^{2}(c-x^{2})^{2} + mx^{2}(x^{2} + mx^{2} - c) + m^{2}(c-x^{2})(x^{2} + mx^{2} - c) - m^{2}x^{4} - m^{3}cx^{2} + m^{3}x^{4} + mcx^{2} - mx^{4} = 0,$$

an identity. Therefore  $\tan \alpha = \tan \beta$ , and  $\alpha = \beta$ . Q. E. D.

[S. M. Barton.]

EXERCISES.

13

A THIN inextensible cord in which the density of the material increases in geometric progression, as the distance from one end increases in arithmetic pro-

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gression is laid directly across a rough horizontal cylinder the circumference of a normal section of which is equal to twice the length of the cord; to determine the co-efficient of friction, supposing the cord to be only just supported when its extremities are in the horizontal plane containing the axis of the cylinder. [William Hoover.]

14

A FAISCEAU of parabolas can be drawn having the pole of a cardioid as the common focus, all passing through one point and all cutting the cardioid at right angles. [H. A. Newton.]

15

It is required to inscribe a rectangle of breadth nD within another rectangle whose diagonal is B. Show that the excess E of the declivity M of the diagonal of the greater rectangle over that of the length of the less may be calculated when n is small, from the formula

$$\sin E = n \cos 2M [1 + 2n \sin 2M + n^2 (3 - \cos 2M) + \dots].$$
 [W. M. Thornton.]

16

ELIMINATE x from the equations

$$x^{6} + ax^{2} + bx + c = 0,$$
  
 $x^{3} + dx^{2} + ex + f = y,$ 

and arrange the result according to powers of y. [Mansfield Merriman.]

17

In HIS work, Die lineale Ausdehnungslehre, ein neuer Zweig der Mathematik, p. 65, Grassmann says: "Lagrange führt in seiner Mec. Anal., p. 14 der neuen Ausgabe, einen Satz von Varignon an, dessen er sich zur Verknüpfung der verschiedenen Principien der Statik bedient. Dieser Satz ist, wie sich sogleich zeigen wird, unrichtig."

In what way is this theorem incorrect as used by Todhunter and others? [Asaph Hall.]

18

A FIXED point of light shines upon a plane which rotates about a fixed axis with a constant angular velocity. A normal to this plane moves at a constant rate (measured in the plane) in the direction of the shadow which the normal casts upon the plane. Given the position of the normal at a time when the light is in the plane; when the light is again in the plane, how far will the normal have moved during the intervening time? [F. P. Leavenworth.]